X-ray-based positioning method for mars entry blackout mitigation

Peng Dang a, Xiaobin Tang a, b, Yunpeng Liu a, b, Junxu Mu a, Sheng Lai a

a Department of Nuclear Science and Technology, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, China
b Key Laboratory of Nuclear Technology Application and Radiation Protection in Astronautics, Ministry of Industry and Information Technology, Nanjing, 210016, China

A R T I C L E   I N F O

Keywords:
X-ray signal
Mars entry blackout
Positioning method
Plasma sheath
Martian atmosphere

A B S T R A C T

Enhancing the positional information acquisition during Mars entry blackout improves the Mars landing mission reliability. A positioning method based on the high-penetration of X-rays was developed to solve the problem. The X-ray signal attenuation was estimated. The positioning performance and the influence of X-ray signal transmission system were also evaluated. Results indicated that the X-ray signal attenuation is extremely low, and the X-ray-based method is expected to be a potential application for obtaining high-precision positional information during Mars entry blackout.

1. Introduction

X-rays have been used in many technology fields since their discovery in 1895. In some space technologies, X-rays also present superiorities (Economou, 2010). Due to the strong penetrability, X-rays in the plasma sheath of the spacecraft communication blackout nearly have no attenuation (Li et al., 2017; Li et al., 2020). When the spacecraft enters into the planetary atmosphere from outer space, a plasma sheath generated by high-speed flight is commonly formed on the entry vehicle surface and disrupts the signal transmission (Kim and Gülhan, 2011; Ramjatan et al., 2017). Mars entry vehicles also experience the process, which is called Mars entry blackout (Levesque and De Lafontaine, 2007; Wang and Xia, 2015). Future Mars missions, such as sample return and manned landing, demand the capability of pinpoint landing at a pre-defined site (Drake et al., 2010; Salotti and Heidmann, 2014). However, most of the sensors cannot be worked under the harsh conditions of Mars entry blackout (Li and Jiang, 2014). Based on the high penetration, X-ray signals are expected to enhance the acquisition of positional information for future Mars landing missions.

The position estimation of all probes which have been successfully landed on Mars is realized primarily with the inertial measurement unit (IMU) based dead reckoning method (Jah et al., 2008; Yu et al., 2017). Nevertheless, previous studies have shown that the positioning error of IMU, even if corrected through other means before the Mars entry blackout occurred, while still exceeds a few hundred meters (Li and Peng, 2011). This increases the failure risk of future Mars pinpoint landing missions. Moreover, the concept, such as Mars Network, which utilizing the transceiver carried by the entry vehicle to receive the radio frequency (RF) signals from beacons or orbiters has been proposed for achieving the increased accuracy in the landing process (Hastrup et al., 2003; Lightsey et al., 2008). Although this concept is capable to reduce the positioning error, the disruption of RF signals during Mars entry blackout still is a critical issue that needs to be resolved.

The aim of this work is to introduce a positioning method which establishes the X-ray signal transmission link between the entry vehicle and orbiters for obtaining the positional information during Mars entry blackout. This paper is structured as followed: Section 2 describes the design and realization of this scheme and introduces the measurement model and X-ray signal transmission link model. In Section 3, the attenuation of X-ray signals in Mars entry blackout is estimated. Moreover, the positioning performance is evaluated and the influence of X-ray signal transmission system on the positioning error is analyzed. Finally, Section 4 contains the conclusion and future research directions.

2. Theories and methods

2.1. X-ray-based positioning method

The strategy using radio beacons to transmit RF signals for the position estimation of Mars entry vehicles has been studied in preceding research. X-ray signals can be utilized to reach the same goal. Part of the signal processing of X-rays is similar to RF signals. The main difference is that the emission and detection of X-ray signals are realized by a modulated X-ray source (transmitter) and an X-ray detector module (receiver), respectively. Furthermore, in the traditional scheme using RF signals, as the technology of RF signals receiving and processing is
relatively mature, each radio beacon just needs to transmit signals and the entry vehicle only receives and processes the signal. However, X-ray detectors cannot identify several simultaneously arrived signals due to the restriction of current technologies. Therefore, the entry vehicle carries modulated X-ray sources and an X-ray detector in this design. The modulated X-ray sources emit the positioning signal to orbiters through the collimation design in different directions. Each orbiter also carries an X-ray detector to receive the signal. Besides, the orbiter which equipped with the data processing center carries a modulated X-ray source to emit X-ray signals. There also has an acquisition pointing and tracking system which maintains the link alignment. The development of above-mentioned technologies and devices has been conducted in a number of XCOM research work, and XCOM experimental test has also been carried out on International Space Station in 2019 (Lori, 2019; Timofeev et al., 2019). Thus, it is available to establish the X-ray positioning signal transmission link between the entry vehicle and orbiters during Mars entry blackout.

The schematic diagram of the proposed method is illustrated in Fig. 1. It is assumed that several orbiters are put into place in advance, which is consistent with the traditional radio beacon based scheme. This method is realized through a six-step process. First, the entry vehicle switches on the X-ray signal transmitter when it reaches the altitude of Mars entry blackout and the time information is transmitted to each orbiter as an X-ray signal. Second, the receiver carried by each orbiter receives the X-ray signal and records the time of signal arrival, and the time of signal emission is obtained by demodulating the received signal. Then the pseudorange between each orbiter and the entry vehicle is calculated by the time difference between the time of signal emission and arrival. The pseudorange represents that the range value is not a true geometric distance between the entry vehicle and orbiters. Next, each orbiter transmits the pseudorange information to the data processing center. Note that the pseudorange information can be transmitted through RF signals between orbiters and the data processing center. The main reason is that the Martian atmosphere is so thin at the altitude of orbiters that Mars entry blackout will not occur, and the RF signal transmission technology is relatively mature. Afterwards, the position coordinates of the entry vehicle can be obtained by the data processing center through the calculation of the received pseudorange information and known orbital parameters. The data processing center also sends the coordinates to the entry vehicle using X-ray signals. In the end, the entry vehicle demodulates the signal containing its position coordinates and thus determines its location.

2.2. Theory of X-ray signal transmission

Predicting the signal attenuation is of great importance for illustrating the superiority of X-rays. The attenuation of X-ray signals during Mars entry blackout are mainly caused by the plasma sheath and Martian atmosphere. The Wentzel–Kramer–Brillouin method (Appendix A) was used to analyse the interaction between X-ray signals and plasma sheath. The electron density of the plasma sheath is a critical parameter that determines the signal attenuation. Fig. 2(a) presents the electron density distribution of the plasma sheath. The stagnation point and the wake region are two areas where the electron density of the entry vehicle surface reaches the maximum and minimum, respectively. The Monte Carlo simulation method was employed to calculate the transmission coefficient of X-ray signals in Martian atmosphere. The interaction between X-rays and Martian atmosphere has been conducted in our preceding work of XCOM (Hang et al., 2019). The previously used atmospheric model was a simplified model which was layered vertically. However, the Martian atmosphere is spherical around the Mars surface. Therefore, a spherical layered model was used in this analysis. The Martian atmosphere was modeled at one layer per kilometre because the atmospheric density varies with the altitude, as shown in Fig. 2(b). The orbiter altitude was fixed at 150 km (Zurek et al., 2015). The entry vehicle altitude was set with a range of 20 km–100 km. The Mars entry blackout occurred in this altitude range according to the data of Mars Science Laboratory (MSL) (Karlgaard et al., 2013). Due to the high-speed flight of the entry vehicle and each orbiter, changes about link elevation angle were also considered. The atmospheric density was estimated by the traditional exponential model (Mazarico et al., 2007).
2.3. Measurement model

The position of the entry vehicle can be estimated by the measurement model. There are four X-ray signal transmission links since that four unknown parameters (the three-dimensional coordinates and the clock offset) are considered here. As shown in Fig. 3, one link is used as examples for description. \( T_o \) is the system time when the positioning signal leaves the entry vehicle. \( T_e \) is the system time when the signal reaches the orbiter. Due to the clock frequency on entry vehicle is difficult to keep same with the ones on orbiters, which easily leads to the offset between the entry vehicle’s clock and system time. \( \delta_t \) is the offset between the orbiter’s clock and system time. The time obtained by the orbiter’s clock can be written as \( T_o + \delta_t \), and the time received by the entry vehicle’s clock can be presented as \( T_e + t \). Several measurement biases \( (\epsilon_p) \) may be caused by the link clutter, such as the ionospheric delay and the tropospheric delay. Therefore, the actual geometric distance \( (r) \) and the pseudorange \( (\rho) \) can be reconstructed as

\[
r = c (T_e - T_o), \tag{1}
\]

\[
\rho = c [ T_o + t - (T_o + \delta_t) + \epsilon_p ] = r + c (t - \delta_t + \epsilon_p), \tag{2}
\]

where \( \delta_t \) can be corrected by monitoring the orbiter and can be regarded as known. \( \epsilon_p \) was treated as a Gaussian-distribution \( N(10, 1) \), whose unit is the meter (Martin, 1978). The simplified assumption can be accepted since that the transmission of X-rays is weakly influenced by the scattering and refraction of the Martian atmosphere.

The pseudorange obtained from the X-ray detector module is transmitted to the data processing center for position coordinates calculation. The coordinates of each orbiter and the entry vehicle are unified to Mars fixed coordinate system. The position coordinates of the entry vehicle are defined by \( (x, y, z) \), and the known position coordinates of Orbiter-\( i \) are given by \( (x_i, y_i, z_i) \). \( r_i \) and \( \rho_i \) can be modeled by

\[
r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}, \tag{3}
\]

\[
\rho_i = f(x, y, z, t) = r + c (t - \delta_t + \epsilon_p), \tag{4}
\]

where \( i \) is the serial number of each orbiter. If the approximate position coordinates of the entry vehicle are known as \( (x', y', z') \) and the corresponding clock offset is \( t' \), then the pseudorange between the approximate and real position can be written as

\[
r_i' = f(x', y', z', t'), \tag{5}
\]

\[
\rho_i = f(x' + \Delta x, y' + \Delta y, z' + \Delta z, t' + \Delta t). \tag{6}
\]

where \( \Delta x, \Delta y, \) and \( \Delta z \) means the deviation between the approximate position and real position at time \( T_o \). Expanding and ignoring the term after the first-order partial derivative. The four signal transmission links satisfy the following formula

\[
\Delta \rho = H \Delta X, \tag{7}
\]

where

\[
\Delta \rho = \begin{bmatrix} \Delta \rho_1 \\ \Delta \rho_2 \\ \Delta \rho_3 \\ \Delta \rho_4 \end{bmatrix}, \quad H = \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & 1 \\ a_{i1} & a_{i2} & a_{i3} & 1 \\ a_{i4} & a_{i4} & a_{i4} & 1 \end{bmatrix}, \quad \Delta X = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z - c \Delta t \end{bmatrix},
\]

where \( a_{i1}, a_{i2}, a_{i3}, \) and \( a_{i4} \) represent the direction cosine of the unit vector pointing from the approximate position to Orbiter-\( i \). Therefore, the coordinates of the estimated position at time \( T_e \) can be derived from the deviation \( \Delta X \) and the approximate position coordinates. Besides, a more accurate solution can be obtained by iterating the above process.

When the entry vehicle emits an X-ray positioning signal at time \( T_o \), the time when the estimated position coordinates are received can be indicated as \( T_o + \Delta t_l + \Delta t_s \) due to the high-speed flight of the entry vehicle. The \( \Delta t_l \) and \( \Delta t_s \) denote the link transmission delay and the signal emission delay caused by the modulation rate of X-ray sources, respectively. Therefore, the pseudorange measurement bias is just one of the positioning error sources, and the time delay from emitting the X-ray positioning signal to receiving position coordinates is also a significant error source. When the position coordinates received can be expressed by \( (x_T, y_T, z_T) \), whose calculation has taken into account the pseudorange measurement bias, the actual coordinates of the entry vehicle can be given as \( (x_{T_o}, z_{T_o}, -\Delta t, -\Delta t, y_{T_o}, z_{T_o}, -\Delta t, -\Delta t, z_{T_o}) \). Thus, the positioning error can be determined by

\[
\varepsilon = \sqrt{P^T P}, \tag{8}
\]

where

\[
P = \begin{bmatrix} x_{T_o + \Delta t_l + \Delta t_s} - x_T \\ y_{T_o + \Delta t_l + \Delta t_s} - y_T \\ z_{T_o + \Delta t_l + \Delta t_s} - z_T \end{bmatrix},
\]

Additionally, the time delay can be obtained by

\[
\Delta t_l = L / V, \tag{9}
\]

\[
\Delta t_s = I / R, \tag{10}
\]

where \( L \) is the link distance; \( V \) is the propagation velocity, which was approximated as the speed of light; \( I \) is the amount of information transmitted in the positioning signal, which was estimated at 100 bits according to the time stamp and position coordinates and \( R \) is the data rate. The other delays are caused by the emission and reception of RF signals, which are negligible because the data rate of RF signal transmission has exceeded several megabits per second (Haskins and Millard, 2010).

2.4. X-ray signal transmission link model

The above analyses have shown that the time delay is an important error source. Therefore, data rate is an important parameter that limits the positioning performance. The signal transmitting power also affects the data rate. The link model was built to evaluate the influence of the X-ray signal transmitting power. The CdZnTe detector was employed as the detection module (Kim et al., 2020). The 4-level pulse-position-modulation method was adopted. The bit-error-rate, signal-to-noise ratio, and signal transmitting power of one signal transmission link can be obtained by (Zhou et al., 2019)

\[
BER = 0.5 \times \text{erfc} \left( \sqrt{\frac{SNR}{2}} \right)
\]
SNR = \left( \frac{\eta_j q P_r / (h \nu)}{2q^2 (\eta_j F_\nu A_r + (P_j \eta_d / h \nu) + I_d / q) + 4KT / R} \right) / R. \tag{12}

P_t = P_s \times \eta_{pla} \times \eta_{atm} \times A_s / \pi (\tan(0.5\omega) L)^2. \tag{13}

where \( P_t \) is the signal receiving power; \( R_t \) is the equivalent impedance; the detection efficiency \( \eta_j \) was more than 90%; and the collection area \( (A_s) \) was 1 m\(^2\) \citep{Quadra,Wei}. The flux of background X-rays \( (F_\nu) \) was 50 m\(^{-2}\)s\(^{-1}\) \citep{Wei}. The divergence angle \( \omega \) was 0.5 mrad \citep{Liu}. The transmission coefficients of the plasma sheath \( (\eta_{pla}) \) and Martian atmosphere \( (\eta_{atm}) \) were determined with the calculation of X-ray signal attenuation. As several positioning signal transmission links are needed to determine position coordinates, the signal transmitting power of the system can be expressed as

\[ P = \sum_{j=1}^{n} P_j. \tag{14} \]

3. Results and analysis

3.1. X-ray signal attenuation

The X-ray signal presents high penetration during Mars entry blackout. The trend of transmission and attenuation coefficients as a function of carrier frequency is shown in Fig. 4. The signal attenuation is considerably affected by the plasma sheath of Mars entry blackout in the RF range. In the stagnation point \( (\eta_{pla}) \) was 1 m\(^2\) \citep{Quadra}. The flux of background X-rays \( (F_\nu) \) was 50 m\(^{-2}\)s\(^{-1}\) \citep{Wei}. The divergence angle \( \omega \) was 0.5 mrad \citep{Liu}. The transmission coefficients of the plasma sheath \( (\eta_{pla}) \) and Martian atmosphere \( (\eta_{atm}) \) were determined with the calculation of X-ray signal attenuation. As several positioning signal transmission links are needed to determine position coordinates, the signal transmitting power of the system can be expressed as

\[ P = \sum_{j=1}^{n} P_j. \tag{14} \]

where \( P_s \) is the signal receiving power; \( R_t \) is the equivalent impedance; the detection efficiency \( \eta_j \) was more than 90%; and the collection area \( (A_s) \) was 1 m\(^2\) \citep{Quadra,Wei}. The flux of background X-rays \( (F_\nu) \) was 50 m\(^{-2}\)s\(^{-1}\) \citep{Wei}. The divergence angle \( \omega \) was 0.5 mrad \citep{Liu}. The transmission coefficients of the plasma sheath \( (\eta_{pla}) \) and Martian atmosphere \( (\eta_{atm}) \) were determined with the calculation of X-ray signal attenuation. As several positioning signal transmission links are needed to determine position coordinates, the signal transmitting power of the system can be expressed as

\[ P = \sum_{j=1}^{n} P_j. \tag{14} \]

3.2. Positioning performance

The scenario of Mars entry blackout was simulated to assess the positioning performance, as shown in Fig. 6. The simulation began at 22:25:00 UTC on Aug 5, 2012 and lasted about 60 s \citep{Israel,Morabito}. The simulation sample step was set to 1 s. The X-ray signal transmission coefficient is higher at large elevation angles, and the positioning accuracy increases with the geometric distribution uniformity of orbiters improve, the orbital parameters are designed with the two factors and presented in the table of Fig. 6. The system visibility can be maintained through the design. The movement of the entry vehicle was reconstructed with reference to MSL \citep{Karlggaard}, whose changes in the trajectory and velocity are shown in Fig. 4, 6. The results are shown in Fig. 7. It indicates the statistical results of the link distance and elevation angle. It should be noted that the link distance includes the X-ray and RF signals transmission links. Moreover, the link distance and elevation angle change over time because of the high-speed flight of each orbiter and the entry vehicle. The upward or downward trend of the curve depends on the relative position between each orbiter and the entry vehicle. In addition, to estimate the position coordinates, the pseudorange was deduced from the signal transmission time delay, which is determined by the link geometric distance as well as by the clock offset and the link clutter. From the simulation model, the geometric distance between the entry vehicle and each orbiter at any sample step can be got through obtaining the coordinates of the entry vehicle and orbiters at any time, as shown in Fig. 7. Moreover, the clock offset of 300 ns was used in the simulation, which is the maximum in the current navigation satellite. We also estimate the time delay caused by the link clutter according to Eq. (2).

The data rate was set to 50 kbps which has been achieved in XCOM, and the bit-error-rate was set to \( 10^{-6} \) to estimate the positioning error \citep{Israel}. The energy of the X-ray positioning signal was set to 70 keV to ensure the X-ray positioning signal transmission link stability \citep{Morabito}. The results are shown in Fig. 8. In the whole simulation process, the maximum positioning error is about 68 m, the minimum is about 27 m, and the mean value is about 43.2 m. Furthermore, there is a trend that the positioning error decreases with the time of the entry vehicle entering the blackout increase. This is because that the entry vehicle speed decreases gradually during Mars entry blackout \citep{Israel}. Therefore, at the same time interval, the entry vehicle displacement in the latter part of Mars entry blackout is less than that in the previous part, resulting in the decrease in positioning error. This indicates that the X-ray-based positioning method is expected to obtain a better positioning accuracy compared with traditional IMU whose positioning
error exceeds a few hundred meters.

### 3.3. Influence of X-ray signal transmission system

The data rate is a critical parameter that limits the signal emission delay. Therefore, analyzing the effect of data rate is beneficial to provide a theoretical basis for the X-ray signal transmission system design. The trend of the positioning error with different data rates is shown in Fig. 9. The positioning error can be considerably reduced by increasing the data rate within a certain range. When the data rate of X-ray positioning signals increases from 10 kbps to 128 kbps, the mean positioning error decreases from 113 m to 37 m. In addition, the maximum and minimum positioning errors also decrease as the data rate increases. This is because increasing the data rate is conducive to reduce the signal emission delay.

---

Fig. 5. X-ray transmission coefficient at different energy levels of (a) 10 keV and (b) 70 keV in Martian atmosphere.

Fig. 6. (a) Simulation scenario created for the positioning performance analysis; (b) Trajectory coordinates and (c) velocity of the entry vehicle.

Fig. 7. (a) Link distance and (b) elevation angle of the signal transmission link between the entry vehicle and each orbiter.
However, the influence of data rate on the positioning error is limited in a higher range since that the data rate is so high that the signal emission delay can be neglected. When data rate increases from 128 kbps to 800 kbps, the positioning error reduction is less than 10 m.

The signal transmitting power also affects the data rate in the X-ray signal transmission system. To obtain the satisfactory positioning accuracy and data rate, the influence of signal transmitting power is also discussed here. The relationship of the mean positioning error and data rate with different levels of signal transmitting power is shown in Fig. 10. The results demonstrate that improving the signal transmitting power within a certain range is an effective means for reducing the positioning error. When the signal transmitting power increases from 21 W to 66 W, the data rate increases from 10 kbps to 128 kbps, and the corresponding mean positioning error decreases from 113 m to 37 m. This is because that increasing the signal transmitting power is beneficial to increase the data rate in the X-ray signal transmission system (Fig. 10). However, the signal transmitting power available to the system is usually restricted due to the power distribution limitation. Thus, the determination of the data rate requires a comprehensive consideration of the influence of positioning error and signal transmitting power. The data rate that can be achieved in XCOM has exceeded 50 kbps, and the corresponding signal transmitting power of this data rate with the four links is about 47 W. It should be noted that the signal transmitting power is not exactly equal to the power needed. The needed power is also related to the technical scheme of the X-ray signal emission source adopted. The electricity demand can be satisfied since that the power source can provide electric power of more than 350 W during Mars entry phase (Ciancetta et al., 2017).

4. Conclusion

This study developed an X-ray-based positioning method for addressing the difficulty of limited access to positional information during Mars entry blackout. This scheme was preliminary analyzed with the simulation method. The results demonstrated that X-rays can be utilized to establish a stable positioning signal transmission link because that the X-ray signal attenuation in the plasma sheath and Martian atmosphere is extremely low. Additionally, this method could maintain a relatively low positioning error. The maximum positioning error of about 68 m, minimum of about 27 m and mean value of about 43.2 m could be achieved if the X-ray signal transmission link with the energy of 70 keV, data rate of 50 kbps, BER of $10^{-6}$ and signal transmitting power of 47 W. Moreover, the positioning error could be effectively reduced by improving the data rate and signal transmitting power within a certain range where the data rate and signal transmitting power do not exceed 128 kbps and 66 W, respectively. These results indicate that the X-ray-based positioning method would be a potential application for enhancing the positional information acquisition during Mars entry blackout in future Mars landing missions.

Although this scheme presents advantages in the mitigation of Mars entry blackout, there still have some aspects that could be strengthened. It may be possible to reduce the number of orbiters, power consumption and obtain more state information if this design can be integrated with other strategies such as IMU and the CubeSat technology. Moreover, the quantitative analysis of the pseudorange measurement errors caused by the effect of the Martian atmosphere on X-ray signal transmission is still needed to study. These issues must be performed in the future research.

CRediT authorship contribution statement

Peng Dang: Methodology, Data curation, Writing - original draft,

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This work was supported by the Aeronautical Science Fund (Grant No. 2018ZC5Z0209), and the State Key Laboratory of Simulation and Effects of Intense Pulse Radiation Environment [Grant No. SKLPR1813].

Appendix A. The Wentzel-Kramer-Brillouin method for signal attenuation calculation in the plasma sheath

The electron density distribution of the entry vehicle surface usually can be approximated as

$$n_e(z) = \begin{cases} n_{max} \exp\left(\alpha_1(z - z_0)^2\right) & (0 \leq z \leq z_0) \\ n_{max} \exp\left(-\alpha_2(z - z_0)^2\right) & (z_0 \leq z \leq z_1) \end{cases} \tag{A.1}$$

where $n_{max}$ represents the peak electron density, which was set with reference to MSL. Additionally, $\alpha_1 = 0.5$, $\alpha_2 = 1$, $z_0 = 0.05$ m and $z_1 = 0.15$ m are parameters to describe the electron density distribution. $z$ is the propagation length. The plasma frequency can be expressed as

$$\omega_p = \sqrt{n_e(z)\varepsilon_0} \tag{A.2}$$

The relative dielectric constant can be written as

$$\varepsilon_r = 1 - \alpha_3^2 / (\alpha_1^2 + \nu^2) - j(\nu / \omega)(\alpha_3^2 / (\alpha_1^2 + \nu^2)), \tag{A.3}$$

where $\nu = 1 \times 10^6$ Hz is the collision frequency; $\omega$ is the angular frequency. The propagation constant is given as

$$k = (\omega / c) \sqrt{\varepsilon_r} = k_0 + jk, \tag{A.4}$$

where $k_0$ and $k$ are the real and imaginary parts of the propagation constant, respectively. The transmission coefficient ($T$) and attenuation coefficient ($Atr$) satisfy

$$T = \exp(-2kz), \tag{A.5}$$

$$Atr = 8.660\pi k_0 |z| \tag{A.6}$$

References


